Increased Nuclear Fusion Yields of Inertially Confined DT Plasma due to Reheat

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The efficiency of energy release has been calculated here for fusion reactions in inertially confined plasmas of high density. It is found that inclusion of reheat due to absorption of the energetic alphas released by the reactions in the plasma itself predicts higher gains G due to ignition. Including losses by bremsstrahlung and fuel depletion we find $G = 71$ for 1 kJ laser energy input with a compression of only 1000 times solid state density.

1. Introduction

The calculation of the nuclear fusion gain G from laser compressed DT plasmas

$$G = \frac{\text{nuclear reaction energy}}{\text{input energy } E_0} \tag{1}$$

is very sensitively depending on the parameters chosen as initial volume V_0, temperature T_0, and density n_0. A further question of essential importance is the model chosen for the slowing down rate of the alphas from the nuclear reactions within the reacting plasma, causing reheat and eventually ignition and self-burning. A further parameter to be included is the radiation loss by bremsstrahlung and the depletion of the nuclear fuel by the reaction itself, while the expansion of the plasma, determined by inertial confinement and adiabatic cooling follows the hydrodynamic equations. The question will not be discussed here, how to release the initial conditions by laser compression of the plasma referring to two standard processes: (a) the gasdynamic ablation-compression resulting in 5% transfer of neodynium glass laser energy into the compressed plasma [1, 2, 3] and (b) the nearly isentropic (therefore more efficient) nonlinear force compression [3, 4, 5] resulting in 50% energy transfer.

The sensitivity of G on the initial condition has been demonstrated even for very simplified gain calculations, where the bremsstrahlung, the reheat and the depletion had been neglected and the numerical calculation of the gain

$$G = \frac{\varepsilon_f}{E_0} \int_0^\infty \mathrm{d}t \iiint_{-\infty}^{+\infty} \mathrm{d}x\, \mathrm{d}y\, \mathrm{d}z\, \frac{n_i^2}{4} \langle \sigma v \rangle \tag{2}$$

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with the ion density of n_i of a 50:50 DT mixture, the reaction energy $\varepsilon_f = 17.6$ MeV and the fusion cross section σ averaged over the thermal velocities of the plasma where the fully hydrodynamic calculation was used including the adiabatic cooling [6]. A little variation of the initial temperature T_0 or of the volume V_0 could result in a drastic change of the gain G. This was the reason why the first examples selected for calculating the gains [8, 9] were so pessimistic, as these were far away from the optimized values [5, 6, 7]

$$G = (E_0/E_{\mathrm{BE}})^{1/3} (n_0/n_s)^{2/3} \tag{3}$$

where the breakeven energy $E_{\mathrm{BE}} = 1.6$ MJ and the solid state density of the DT mixture

$$n_s = 5.8 \times 10^{22} \text{ cm}^{-3} .$$

The further condition had to be fulfilled that an optimum initial temperature $T_{\mathrm{opt}} = 10.6$ keV had to be selected. These conditions are algebraically identical with the later formulation

$$G = C\, n_0\, R_0 \tag{3a}$$

which Kidder [10] derived for inertial confinement to substitute the Lawson criterion for statically magnetic confinement, where R_0 is the initial pellet radius and $C = 1.66 \times 10^{-22}$ cm^2 from our values of Eq. (3), about 1.5 times higher than Kidder's [10] value and a factor two less than Brueckner's [17] value and comparable with the very indirectly derived values from Nuckolls' [1] hydrodynamic numerical calculations. While the calculations leading to the Eqs. (3) and (3a) have not included the reheat by the reaction products, we present the contribution of these reheat effects on the numerical values of the reaction gain G as functions of the initial parameters e.g. the ion density n_0, plasma volume V_0, plasma temperature T_0 etc. for the case of DT fusion. The selection of optimum conditions is in this case more difficult. The problem of reheat is involved with the very complex question, what penetration depth for the MeV alphas in the high density plasma is to be taken. We have calculated it on the basis of a modification of the well known Bethe-Bloch formalism used in nuclear physics which one may call the "collective model". In Section 2 this aspect is discussed. Then in Section 3 we have sketched how to compute G with this reheat aspect and ionic depletion under inertial confinement. Finally in Section 4 we discuss the numerical values obtained.

2. Collective Model for Reheat

The heavy charged particles created in the fusion reactions have usually the initial energy of the order of several MeV whereas the plasma has the temperature in the range of keV. Initially the heavy particles therefore moves in a straight line from the point in the plasma where it is created until it loses energy by collisions with the plasma electrons to the magnitude of average thermal energy. From then on it takes part in the random motion of the plasma. This distance is its range. To evaluate it we have used the following modification of the Bethe-Bloch formalism [11]. The energy loss per unit path can be written as

$$\frac{\mathrm{d}E}{\mathrm{d}x} = 4\pi n_e \frac{Z^2 e^4}{m_e v^2} \ln\left(\frac{b_{\max}}{b_{\min}}\right)$$

where E is the energy of the heavy particle, Z_e its charge, n_e the plasma electron density and $b_{\max}$ and $b_{\min}$ are the maximum and minimum value of the impact parameters.

Now in the plasma one has a natural cut-off due to the Debye screening so that one can set $b_{\max} = \lambda_d$ where λ_d is the Debye radius: $\lambda_d = \sqrt{kT/4\pi n_e e^2}$. If one now chooses for $b_{\min}$ the classical value $b_{\min} = z e^2/m_e v^2$ one obtains the following expression for the range R_α

$$R_\alpha = \frac{e^2}{2kT} \frac{m_H}{m_e} \mathrm{Ei}(\ln(\lambda E_0^2)) \tag{4}$$

where m_H is the mass of the heavy particle, E_0 its initial energy,

$$\lambda = \left(\frac{m_e}{m_H}\right)^2 \frac{kT}{\pi n_e z^2 e^6}$$

and the integral logarithm

$$\mathrm{Ei}(x) = \int_{-\infty}^{x} \frac{e^t}{t} \mathrm{d}t$$

had been used.

The inclusion of radiation emitted during the slowing down process of the alphas results in a negligible contribution [12] similar to the case of the slowing down on the basis of a Fokker-Planck mechanism [13, 14].

3. Calculation of the Fusion Gain

We base the computation of G on the model of a spherical plasma with initial temperature T_0, radius R_0 and ion density n_i^0 expanding symmetrically in vacuum. This self-similarity model has also been used by Basov and Krokhin [8], by Dawson [9] and has been justified from the hydrodynamic equations [15] by averaging densities or by a kinetic equation [7] or by the classical case of a Gaussian density profile [16].

Now the equations of motion of a spherical plasma are obtained firstly by equating the work done by the pressure to the drop in temperature and also secondly from the energy conservation where the reheat due to absorption of the charged particles released in the reactions is to be taken into account. One thus obtains

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\bar{M} \frac{\dot{R}^2}{2}\right) = P \frac{\mathrm{d}V}{\mathrm{d}t} \tag{5}$$

where P is the pressure, $\bar{M}$ an average mass for the plasma and V the volume of the plasma at time t. One has

$$V = \frac{4}{3}\pi R^3 \text{ and } P = 3(N_i + N_e)\frac{kT}{4\pi R^3} \tag{6}$$

where N_i and N_e denote the total number of ions and electrons in the plasma. If one assumes that the Plasma has to increase linearly on the radius from the centre to the boundary (self-similarity model [8, 11]) one has $\bar{M} = \frac{3}{5} M$ where M is the total mass of the plasma i.e. $M \simeq m_i N_i$ where m_i is the average ion mass. Now due to the reactions the electrons do not charge so that one can write $N_i = \bar{z} N_i^0$ where $\bar{z}$ is the average ionic charge and N_i^0 the total number of initial ions present. Now the alphas produced in the reactions will be mostly absorbed for high density so that although the species of the ions change their total number is approximately constant

$$N_i + N_e \cong (1+\bar{z})\, N_i^0 .$$

One then obtains utilising (5) and (6)

$$kT = \frac{m_i}{5(1+\bar{z})} R\ddot{R} \tag{7}$$

where the dot denotes the derivative with respect to the time t. Now it is useful to introduce the substitution

$$R_0/R = \cos\theta \tag{8}$$

where the parameter θ is then a function of t.

One then obtains in terms of θ

$$\frac{d\dot{\theta}}{d\theta} = \frac{T}{T_0}\,\frac{\dot{\theta}_0^2 \cot\theta\cos^2\theta}{\dot{\theta}} - \dot{\theta}\cot\theta\,[\sec^2\theta + \tan^2\theta] \tag{9}$$

where the initial values for $t=0$ are given by

$$\left(\frac{d\dot{\theta}}{d\theta}\right)_0 = 0 \quad \text{and} \quad \dot{\theta}_0^2 = \frac{5(1+\bar{z})\, kT_0}{m_i R_0^2}$$

assuming $\dot{R}_0 = 0$ i.e. the plasma starts expanding from rest.

Now we introduce the probability P_α for the absorption of an alpha particle by the plasma itself as approximately given by

$$P_\alpha = R/(R+R_\alpha)$$

where R_α denotes the range according to Figure 4. Let $\varepsilon_{F,\alpha}$ denote the part of the fusion energy released carried by the alpha. The equation of conservation of energy is then

$$\frac{d}{dt}\left(\tfrac{3}{2}(N_i+N_e)\,kT\right) = -\frac{d}{dt}\left(\frac{1}{2}\bar{M}\left(\frac{dR}{dt}\right)^2\right) + \varepsilon_{F,\alpha}\frac{R}{R+R_\alpha} \times \frac{n_i^2\langle\sigma v\rangle}{A}\,\frac{4}{3}\pi R^3$$

which then in terms of θ is

$$k\frac{dT}{d\theta} = -2\,kT\tan\theta + \frac{2}{3}\,\frac{\varepsilon_{F,\alpha}}{1+\bar{z}} \times \frac{1}{1+\dfrac{R_\alpha}{R_0}\cos\theta}\,\frac{n_i^2\langle\sigma v\rangle}{n_i^0 A\cos^3\theta}\,\frac{1}{\dot{\theta}} . \tag{10}$$

Finally we need the equation for the change of the ion density n_i in terms of θ to describe the depletion effect which is given by

$$\frac{dn_i}{d\theta} = -\frac{1}{2}\,\frac{n_i^2\langle\sigma v\rangle}{\dot{\theta}} - 3n_i\tan\theta . \tag{11}$$

The first term here corresponds to the reaction and the second to the radial expansion of the plasma.

The expression for G then simplifies to

$$G = \frac{\varepsilon_F}{E_0} V_0 \int_0^{\pi/2} \frac{n_i^2\langle\sigma v\rangle}{A\cos^3\theta}\,\frac{d\theta}{\dot{\theta}} .$$

Here $\langle\sigma v\rangle$ depends on θ through the temperature T as $T = T(\theta)$. This integral can be evaluated only numerically utilising the infinitesimal variations

$$n_i(\theta+\Delta\theta) = n_i(\theta) + (dn_i/d\theta)_\theta\,\Delta\theta ,$$
$$T(\theta+\Delta\theta) = T(\theta) + (dT/d\theta)_\theta\,\Delta\theta ,$$
$$\dot{\theta}(\theta+\Delta\theta) = \dot{\theta}(\theta) + (d\dot{\theta}/d\theta)_\theta\,\Delta\theta$$

where the derivatives are calculated according to (9), (10) and (11). The initial temperature T_0 is related to the input laser energy E_0 according to

$$E_0 = 2\pi(1+\bar{z})\, n_i^0 R_0^3 T_0 .$$

The numerical results are discussed in the next section.

4. Numerical Results

Firstly we note that the inclusion of Bremsstrahlung numerically does not alter the gains values with reheat appreciably for the cases considered in the following cases for DT. Numerically it has been found that if one chooses the initial parameters carefully one can attain the situation where although the initial temperature is a few keV the reheat contribution overrides the adiabatic cooling due to expansion — this is the case if "self-ignition" which is of interest.

As we arrive at sufficiently high values of fusion gains for laser-fusion reactors, we concentrate on the case of a compression of the plasma to 1000 time of the solid state density $n_0 = 10^3\, n_s$. Figure 1 describes the gains G depending on the energy E_0 transferred to the compressed plasma in its initial state for various initial volumes V_0. The fully drawn curves are without reheat, without depletion and without bremsstrahlung. The envelope of these curves corresponds to the optimum values given

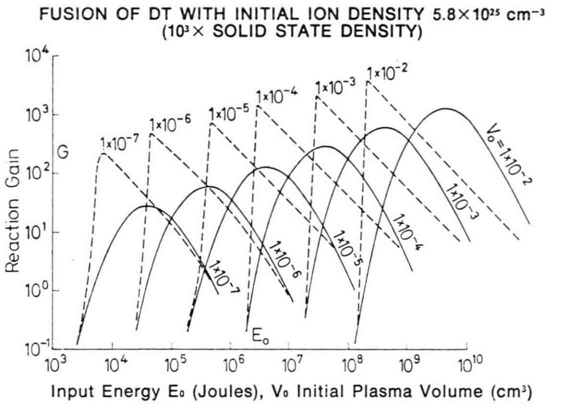

Fig. 1a. Nuclear fusion gain G for 50:50 DT plasma for various initial densities as multiples of the solid state density n_s depending on the energy E_0 transferred into the compressed plasma and the initial solid state volume V_s before compression. The calculations include reheat, fuel depletion and losses by bremsstrahlung.

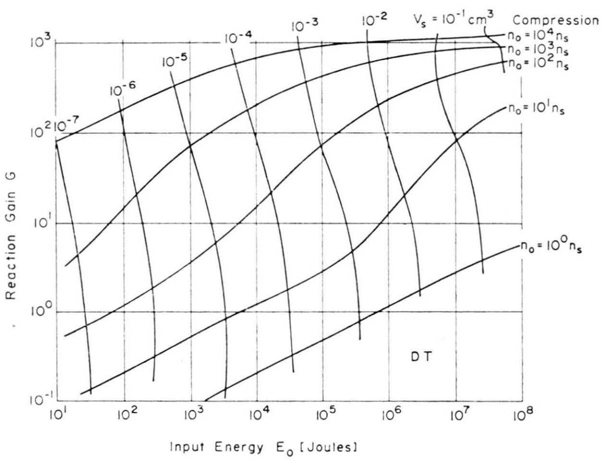

Fig. 1b. Envelopes of dashed curves of Fig. 1a, however, including a small correction due to full inclusion of bremsstrahlung losses for various initial densities n_0 given in multiples of the solid state density n_s. The volume V_s corresponds to the solid state (i.e. before compression); $V_s = V_0 = V_0 n_s/n_0$.

by Equation (3). Inclusion of reheat and depletion of full results in the dashed curves. Their envelope is a nonlinear plot and cannot be described by an analytic formula like Equation (3). Including bremsstrahlung, we arrive at envelopes as given in Fig. 1a for various n_0 and $V_s = V_0 n_s/n_0$.

What is surprising, is that the initial energies E_0 for the maximum gains correspond to 5 keV per particle at E_0 near 1 kJ and even 1 keV only at E_0 near 1 MJ. These very much less energies than the usual 10 keV necessary for DT fusion when calculated without reheat can be understood, when the time dependence of the temperature is calculated. In Fig. 2 we see these cases for an initial volume of 10^{-3} cm³ and a set of initial energies E_0 of 1.07, 1.6 and 2.7 MJ. The case of the lowest energy shows that the reheat is nearly keeping a constant temperature despite the adiabatic cooling of expansion. The higher energies show the ignition and self-burning, raising the temperature above 100 keV after some picoseconds. The initial temperatures explain why gains exceeding 1000 can be reached by self-burning. The low ignition temperature is possible only at very high densities (see Fig. 1b) with the necessarily high collision frequencies.

The inclusion of bremsstrahlung losses was done by reducing the temperature at each increment $d\theta$ (corresponding to time steps of about 10^{-13} seconds) by assuming that the bremsstrahlung is being emitted without reabsorption in the plasma. Even by this pessimistic assumption the results of Fig. 1b show a minor change only against the case without bremsstrahlung losses (Figure 1a). This was the experience also in other calculations [17]. We consider (Fig. 1b) the following cases as examples:

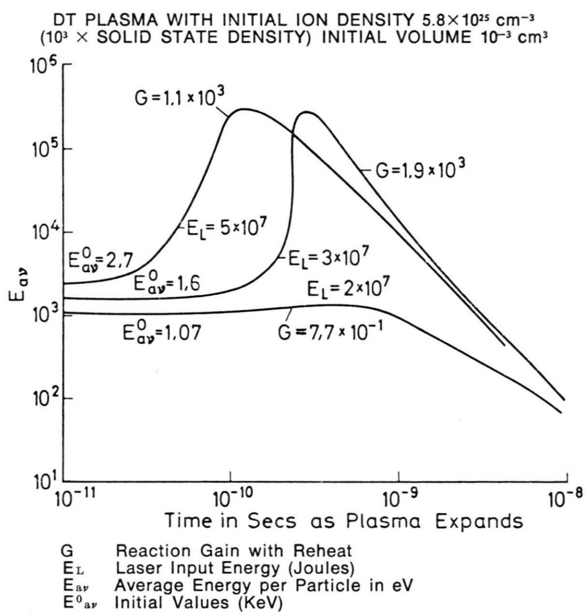

Fig. 2. Time dependence of the temperature of a DT laser plasma for compression 10^3 times solid, initial volume 10^{-3} cm³ and varying initial energy E_0. A strong increase of the gain results by a small increase of E_0 when ignition and self-burning is happening.

1. $E_0 = 1.0$ kJ; $n_0 = 10^3\, n_s$; $V_0 = 10^{-8}$ cm^3; $V_s = 10^{-5}$ cm^3; initial average particle energy 5.38 keV, corresponding to an initial temperature of 3.58 keV; ignition starts at 3.4 psec; while without bremsstrahlung losses we arrive at a gain of 83.8 (Fig. 1), we find with the losses as the most general value a *total gain* $G = 71$.

2. $E_0 = 7.0$ kJ; $n_0 = 10^3\, n_s$; $V_0 = 10^{-7}$ cm^3; $V_s = 10^{-5}$ cm^4; initial average particle energy 3.7 keV corresponding to a temperature of 2.46 keV; ignition starts at 9.8 psec; gain without bremsstrahlung: 220.2 *total gain G including all processes* = 178.

These examples were selected to demonstrate the strong increase of the gain with the collective reheat claculation over the earlier calculations [6, 7, 10] following Eq. (3) or (3a). Using our constants [6] which were within the values of the other mentioned authors, we arrive from the earlier calculations for $n_0 = 10^3\, n_s$; $E_0 = 1$ kJ at $G = 8.54$ (or for $E_0 = 7$ kJ at $G = 16.3$) which is remarkably less than our values 1. and 2. which correspond to the values necessary for laser-fusion reactors. The parameter of transferring 1 kJ laser energy to a compressed plasma is about the state of the art, especially if the nonlinear force compression is applied [3, 4, 5].

Starting from 50 atm gas, compressions exceeding a factor 1000 have been reported by Basov [18]. The gains, however, are very sensible against the selection of the pellet parameters, to which determination our calculations may offer a new basis.

The agreement of our analytical model with the very detailed numerical calculations of Nuckolls [1] can be seen from the following example: Nuckolls needs 95% of his incident laser energy of 200 kJ for ablation, which corresponds to a value of 40 for his definitions of the gain at 10^4 times compression. Our corresponding $E_0 = 10$ kJ arrives at a gain of 660 in our definitions (Fig. 1b) corresponding to a gain of 33 in the definition of Nuckolls. This agreement justifies our assumptions for the reheat calculations, as Nuckolls used reheat values semi-empirically derived from nuclear reactions [19]. Our assumptions are straightforward and do not involve questionable restrictions. The resulting gains, however, are even more sensibly dependent on the right selection of E_0, V_0 and compression than before due to the ignition (Figure 2).

5. Conclusions

Calculation of the nuclear reaction gains of 50:50 DT plasmas of only 10^3 times solid state density including the exact gasdynamic behaviour with adiabatic cooling, losses by bremsstrahlung, depletion of nuclear fuel by burning and reheat based on a collective model arrive at fusion gains of 71 and 178 for energy input of 1 kJ and 7 kJ respectively. These gains can be assumed as sufficient for a pellet power reactor. It is therefore possible to work with compression of 1000 times of the solid state only and with energies for laser which correspond to the present state of the art. Relative good agreement of our analytical model exists with the semi-empirical computation of Nuckolls.

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